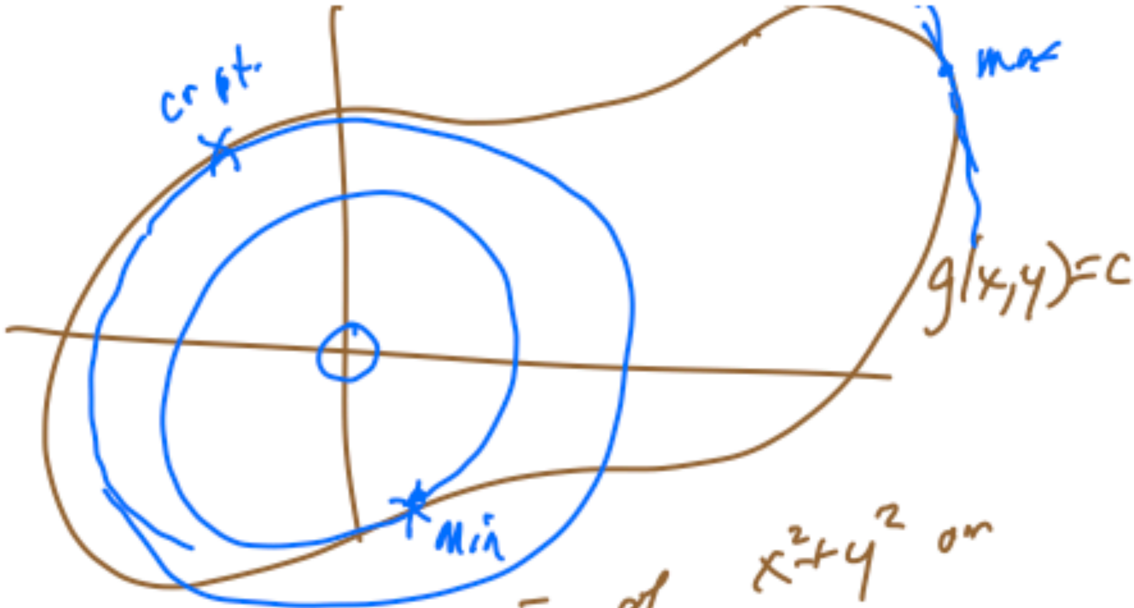


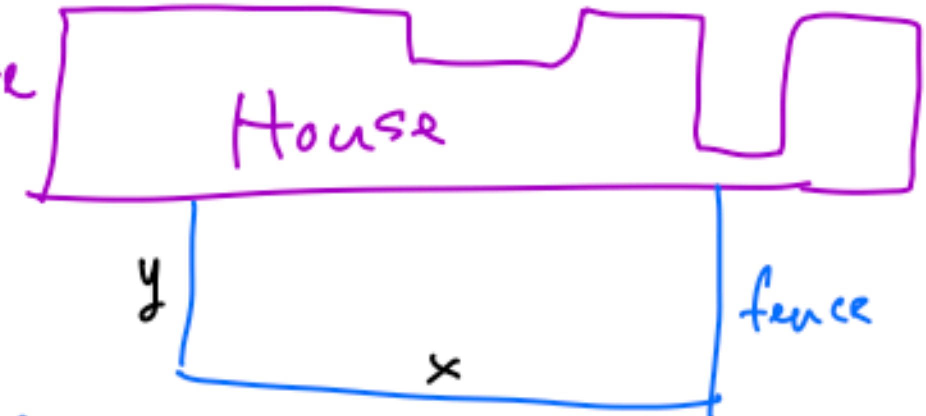
find max & min of x^2+y^2 on $g(x,y)=c$.

→ Look for where contours (level sets) of x^2+y^2 become tangent to the curve.

$x^2+y^2=c$

Example You have 100 ft of chain link fencing to make a rectangular area attached to your house.

Picture



How do we design the rectangular area so that we enclose the maximum area?

Constraint $\overset{g(x,y)}{x+2y} = 100$ (feet) $x, y > 0$

Function $f(x,y) = xy$

always the quantity that you want to maximize or minimize.

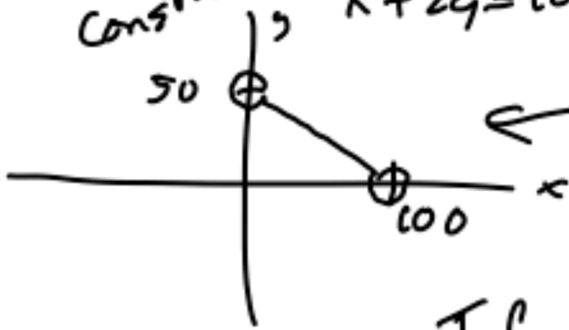
$\nabla f = (y, x) \qquad \nabla g = (1, 2)$

$\nabla f = \lambda \nabla g, \quad g = 100$

$$\left.\begin{array}{l} y = \lambda \cdot 1 \\ x = \lambda \cdot 2 \\ x + 2y = 100 \end{array}\right\} \quad \begin{array}{l} y = \lambda \\ x = 2y \\ 4y = 100 \\ \boxed{\begin{array}{l} y = 25\text{ ft} \\ x = 50\text{ ft} \end{array}} \end{array}$$

1 cr pt.

Constraint $x + 2y = 100$, $x, y > 0$



since endpoints not included, not compact.

If we do include the endpoints $(0,50), (100,0)$

$f(x,y) = xy = 0$

Then compact.

(x,y)	$f(x,y)$
$(50,25)$	1250 sq ft.
$(0,50)$	0
$(100,0)$	0

global max area

Calc I solution to this problem:

maximize $f(x,y) = xy$

$$x + 2y = 100$$

$$x = 100 - 2y$$

$$F(y) = (100-2y)y = 100y - 2y^2$$

$$0 \leq y \leq 50$$

$$F'(y) = 0 = 100 - 4y \Rightarrow y = 25$$

$$x = 100 - 2y = 100 - 2(25) = 50.$$

endpoints $y=0, y=50 \Rightarrow F(y) = 0.$

$\therefore$ $y = 25$ is global max

Fence dimensions are $\boxed{\begin{array}{l} x = 50 \\ y = 25 \end{array}}$.